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Relativistic Covariance of the κ-Poincaré Model with Multiple Causally Connected Interactions

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Introduction

The κ-Poincaré model [2] is a Hopf algebra-based deformation of Special Relativity that features a modified dispersion relation, modified momentum conservation law, and relative locality effects. Until recently, the covariance of the model was understood only partly [2, 3]. Here we present our result [1] that the Hopf algebra structure can be used to lift all symmetries to the multi-particle phase space in a non-trivial way, leading to a covariant theory that describes arbitrarily many interacting particles.

The κ-Poincaré Hopf Algebra

In the bicrossproduct basis of the $(1+1)$-dimensional κ-Poincaré Hopf algebra the generators associated to spacetime translations, P_0, P_1, and boosts, N, satisfy the following algebra and coalgebra

$$[P_0, P_1] = 0\,, \qquad [N, P_0] = P_1\,, \qquad [N, P_1] = \frac{\kappa}{2}\left(1 - e^{-2P_0/\kappa}\right) - \frac{1}{2\kappa}P_1^2\,,$$

$$\Delta(P_0) = P_0 \otimes 1 + 1 \otimes P_0\,, \qquad \Delta(P_1) = P_1 \otimes 1 + e^{-P_0/\kappa} \otimes P_1\,,$$

$$\Delta(N) = N \otimes 1 + e^{-P_0/\kappa} \otimes N\,.$$

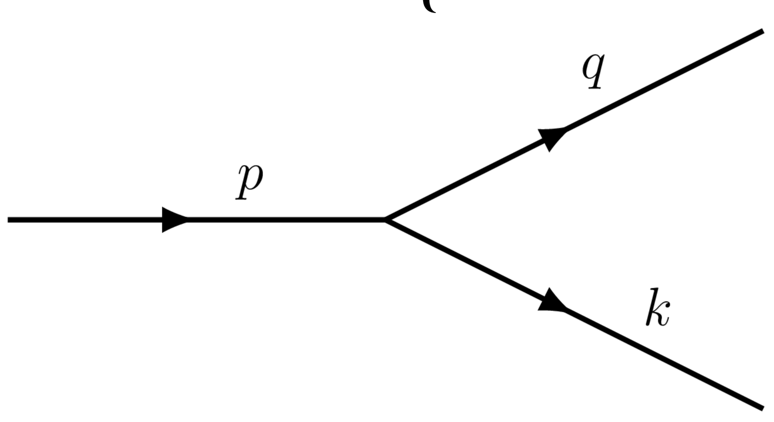
The parameter κ, with dimensions of a momentum, governs the deformation with respect to the classical Poincaré algebra, which is recovered in the $\kappa^{-1} \to 0$ limit. Because of the connection to quantum gravity research, the parameter κ is expected to be roughly of the order of the Planck scale $E_p \simeq 10^{28}$ eV.

To obtain a free-particle model that is invariant under the imposed symmetry algebra, the generators P_0, P_1, N are represented as functions on phase space and the Casimir of the algebra, $C = 4\kappa^2 \sinh^2\left(\frac{p_0}{2\kappa}\right) - (p_1)^2 e^{p_0/\kappa}$, is used both as dispersion relation $C = m^2$ and as Hamiltonian. With this setup, both the dynamics and the dispersion relation are automatically invariant under transformations generated by P_μ and N, and in the limit $\kappa^{-1} \to 0$ Special Relativity is recovered.

Covariance at an Interaction Vertex

The coproduct Δ induces a non-trivial momentum conservation law $p = q \oplus k$ in an interaction $p \to q + k$, namely [2]

$$(q \oplus k)_\mu \equiv (\Delta(P_\mu))(q, k) \quad \text{i.e.,} \quad \begin{cases} (q \oplus k)_0 &= q_0 + k_0\,, \\ (q \oplus k)_1 &= q_1 + e^{-q_0/\kappa} k_1\,. \end{cases}$$



Covariance requires that the conservation law have the same form in any inertial frame. In particular, boosts must leave the conservation law invariant. However, the (infinitesimal) action of a boost of rapidity ξ on a particle's momentum, given by $N^\xi \triangleright p = p + \xi\,\{N, p\}$, satisfies [2]

$$p = q \oplus k \quad \Rightarrow \quad N^\xi \triangleright p = (N^\xi \triangleright q) \oplus (N^{\xi \triangleleft q} \triangleright k), \qquad (1)$$

where the rapidity acting on the *second* outgoing particle receives a *backreaction* $\xi \triangleleft q = e^{-q_0/\kappa}\xi + \mathcal{O}(\xi^2)$ from the first outgoing momentum q. Thus, the requirement for covariance demands a non-trivial implementation of boosts on multi-particle phase space.

Generators Lifted to Multi-Particle Phase Space

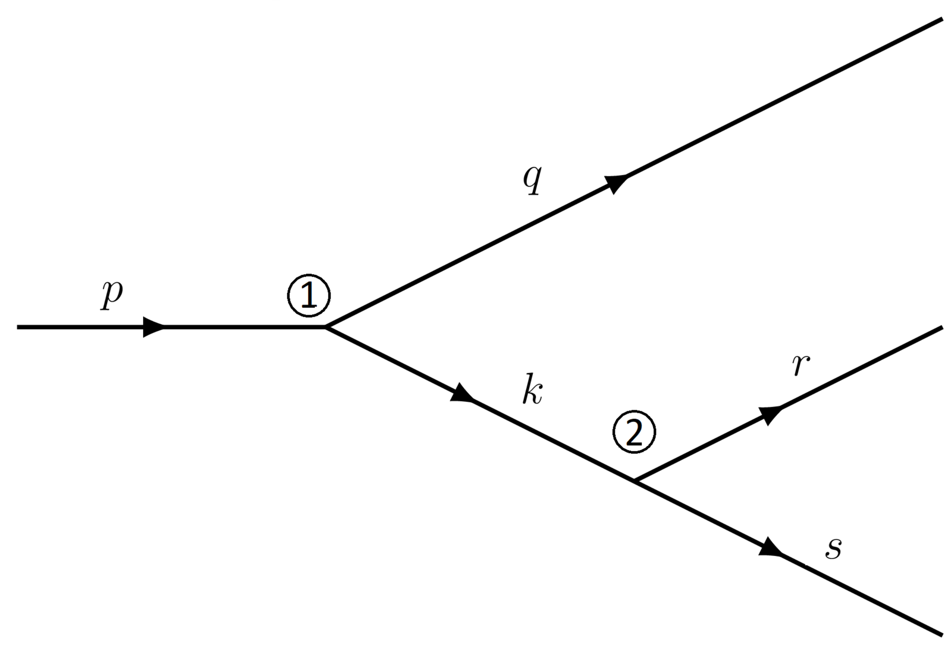
As $(q \oplus k)_\mu$ is interpreted as the *total momentum* of the 2-particle system after the interaction, it is natural to interpret it also as the generator of translations, acting on 2-particle phase space (see also [3]). From this point of view, Δ also induces a total boost generator on 2-particle phase space:

$$N_{q\oplus k} \equiv (\Delta N)(q, k) = N_q + e^{-q_0/\kappa} N_k\,,$$

N_q and N_k being the single-particle boosts corresponding to q and k. Eq. (1) can then be written in a way that makes covariance manifest:

$$\boxed{p = q \oplus k \quad \Rightarrow \quad N_p^\xi \triangleright p = (N_{q\oplus k}^\xi \triangleright q) \oplus (N_{q\oplus k}^\xi \triangleright k).}$$

In general the total boost generator is given by the (ordered) Δ-induced sum of the individual boost generators of *all* causally connected particles present at a given instant. E.g. for the process $p \to q + k$ followed by $k \to r + s$ the total boost generator, after both interactions, reads $N = N_q + e^{-q_0/\kappa} N_r + e^{-(q_0 + r_0)/\kappa} N_s$.



The conservation law is only one aspect of the model. However [1]:

> *When symmetries are implemented via the Hopf algebra-induced lift, all aspects of the κ-Poincaré model behave covariantly.*

Conclusion and Discussion

In the κ-Poincaré model the underlying Hopf algebra structure provides a way, via the coproduct Δ, to lift symmetry transformations from the 1-particle phase space to any relevant multi-particle phase space in such a way that the interacting theory remains covariant. This raises the question: to what extend is this result particular to κ-Poincaré and to what extend is it a general consequence of the underlying Hopf algebra structure?

[1] G. Gubitosi and S. Heefer, "Relativistic compatibility of the interacting κ-poincaré model and implications for the relative locality framework," *Phys. Rev. D* **99** (Apr, 2019) 086019.

[2] G. Gubitosi and F. Mercati, "Relative locality in κ-poincaré," *Classical and Quantum Gravity* **30** (2013), no. 14, 145002.

[3] G. Amelino-Camelia, M. Arzano, J. Kowalski-Glikman, G. Rosati, and G. Trevisan, "Relative-locality distant observers and the phenomenology of momentum-space geometry," *Classical and Quantum Gravity* **29** (Apr., 2012) 075007, 1107.1724.